Dennis Liu

Co-Head of Research - EUTIC

Viet Hung Ha

Co-Head of Research - EUTIC

Rebecca Collins

Macro Research Analyst - EUTIC

Shreyas Saxena

Macro Research Analyst - EUTIC

Harvey Wynn

Macro Research Analyst - EUTIC

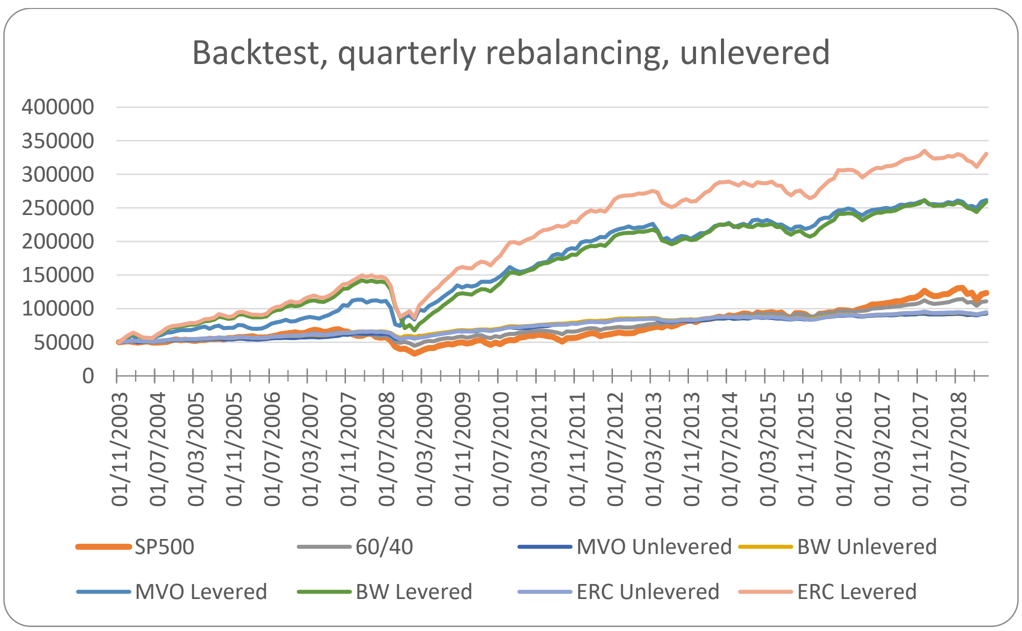
Exploring asset allocation strategies: an introduction to risk parity

EXECUTIVE SUMMARY

In this paper we give a brief introduction to return-free asset allocation strategies, namely the risk parity framework. The intuition is to choose asset classes such that the portfolio will be minimally affected by extrinsic factors, and determine corresponding weightings through variance-covariance of the asset classes exclusively. We then consider two approaches, one based on that of Bridgewater Associates, and the other based on equal risk contributions for all asset classes.

	Corporate Credit	*EM Credit*	*IL Bonds*	*Commodities*	*Nominal bonds*	*Equities*
BW allocation	0.0905	0.1334	0.3448	0.0780	0.2348	0.1185
ERC allocation	0.1960	0.1255	0.2445	0.0999	0.1820	0.1521

Through heuristic and numerical means, we obtain the above table which gives us optimal weightings for each of the six asset classes chosen under both approaches. Using these allocations, we then perform back-tests to illustrate possible portfolio performances for risk parity strategies, compared to more traditional approaches. The results indicate that the volatility for both risk parity portfolios are much lower, and they are more resilient to external conditions. We also see that by leveraging both portfolios to match annualised S&P 500 volatility, both outperform by a considerable margin.



1 INTRODUCTION

1.1 MOTIVATION

Traditional asset allocation strategies have centred around two themes, namely risk and return, with an emphasis on the latter. However, recent events, such as the 2008 financial crisis, have shown that this framework is not infallible. Thus, in this paper, we will explore an alternative form of asset allocation strategies, the risk parity framework, which will focus exclusively on the risk profile of different assets.

The first risk parity fund was started by Bridgewater Associates' All-Weather Fund in 1996, although the basis for risk parity has been developed since the 1950s with Markowitz's mean-variance portfolio allocation strategy. The basic desire of the founder of Bridgewater Associates, Ray Dalio, was to create a portfolio allocation that could perform well in different economic environments. As a concept, risk parity took time to gain traction in the asset management industry, partly due to the reliance of leverage as a part of the strategy, as the conventional wisdom of the time was the leveraging was risky, and partly due to peer risk holding back investors from trying new investment strategies. The first success of risk parity came to fruition after the March 2000 crash, whereas previously, equity-heavy portfolios outperformed risk parity portfolios. Over time, the concept started to gain in interest following over a decade of poor performance from traditional portfolios, first being used in a white paper by Edward Qian in 2005. Following the 2008 financial crisis, interest in risk parity funds increased substantially, and ever since mainstream asset managers have been rolling out risk parity funds for investors.

Since the All-Weather strategy was first launched, risk parity funds now have at least $100bn invested in them. Risk parity has been particularly popular with institutional investors, with one of the largest Canadian pension funds using risk parity for its whole plan, according to All-Weather. 46% of institutional investors are currently using risk parity strategies as a part of their broader strategy, with an additional 8% seeking to, according to a survey from Chief Investor Officer magazine in 2014.

1.2 THE INTUITION OF RISK PARITY

Although the term 'Risk Parity' is a broadly used term, covering many allocation strategies using various intuitions, the basis of Risk Parity is Risk Contribution. Risk, defined as the volatility of an asset, is evenly balanced across a portfolio through the contribution it makes towards the total risk of a portfolio.

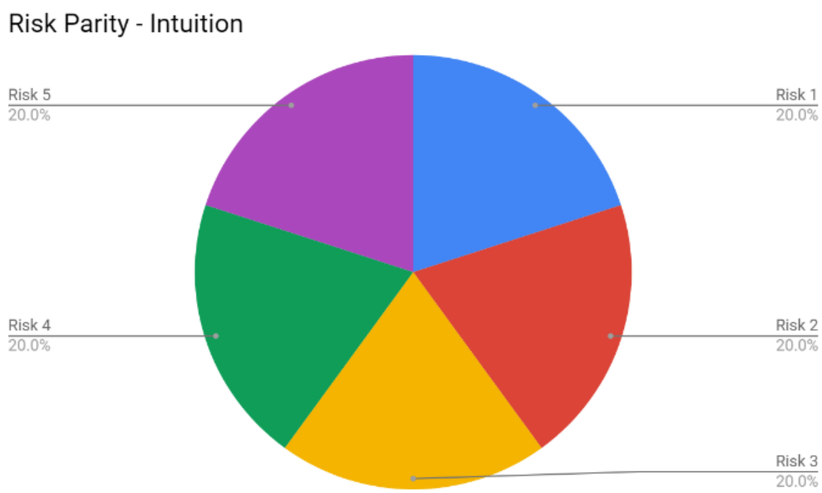


Figure 1.2.1. The intuition behind equal risk allocation of a 5-asset portfolio. Note that weightings are allocated based on risk contribution and not nominal amounts.

Figure 1 shows how a 5-asset portfolio should look in regards to risk contribution. In reality, the nominal weighting of each asset will vary on account of the fact that some assets with be riskier than others, and so will have a weight corresponding to its volatility.

1.3 ADVANTAGES OF RISK PARITY

The basic intuition posits that the conventional portfolio is not truly risk-balanced. Consider the weighting of a 60/40 portfolio allocation, compared to the risk contribution of each asset class. Despite making up 60% of the total capital allocation in the portfolio, equity accounts for 90% of risk contribution, showing that the portfolio weighting does not actually balance risks across asset classes, and that most traditional portfolios tend to be overexposed to the volatile and cyclical nature of equity markets. Therefore, a risk parity portfolio better balances the risks across the portfolio, theoretically delivering a better Sharpe Ratio, achieving a lower level of risk while offering similar or better rates of return. Furthermore, compared to the Markowitz (mean-variance) approach of asset allocation there are a few key differences:

1. Diversification: One of the principal concerns with the Markowitz model is the lack of diversification of optimal portfolios. As Braga indicates, "Efficient allocations frequently completely exclude some asset classes of the chosen investment universe and give extremely large weights to some others"[1]; mean-variance optimised portfolios only consider the overall portfolio risk. Furthermore, with regards to the Efficient Frontier in the Markowitz model, the most north-east point along this frontier (i.e. the optimal portfolio with highest risk and highest return) usually appears to be concentrated on a singular asset. This is primarily due to Markowitz's methodology putting significant importance on the expected returns of the asset classes over their variances of correlations, and thus are often the ones most likely to suffer from large estimation errors[2]. Hence, in essence, the mean-variance optimizers are just maximizing the estimation error. Risk parity circumvents this problem by focusing solely on asset risk, disregarding expected returns altogether.
2. Stability of Portfolio: Mean-variance optimized portfolios are generally unstable due to the high sensitivity of weightings to small changes in the "estimated parameters". This could be worse in situations when groups of asset classes have similar risk-return profiles as this may cause changes as to which asset classes should or should not be more dominant in the portfolio. This can lead to portfolios based on this model to largely underperform under certain economic environments compared to other portfolios at a similar risk level. Risk parity portfolios focus on diversifying the risk profiles through equalising risk contributions, as well as selecting asset classes with varying performances under different economic scenarios.
3. Estimation Error: Mean-variance optimizers in general can produce a single unique portfolio that is supposed to outperform all others for a certain level of risk. This is inaccurate, however, as this assumption of true uniqueness lies upon the incorrect

[1] Braga, p.10

[2] Braga

assumption that the input parameters have no estimation error and are the correct parameter values. By focusing solely on variance, risk-parity models reduce this exposure by reducing the number of parameters to be estimated.

4. Out-of-Scope Performance: We see poor risk-adjusted and low expected performance of mean-variance optimized portfolios when we test the portfolio outside the time period used to estimate the parameters. Risk parity portfolios aim for good risk-adjusted performance by choosing asset classes such that portfolio performance is minimally affected by extrinsic factors, and focuses solely on asset classes outperforming cash over time.

1.4 Methods

In this paper we will focus on two risk parity models, the Bridgewater (BW) approach and the equal risk contribution (ERC) approach.

BW

The basic intuition of the Bridgewater method lies in allocating risk based on two key macroeconomic variables: inflation and economic growth. From here, assets are equally allocated into the four possible environments an investor may experience at any given time, shown in the figure below.

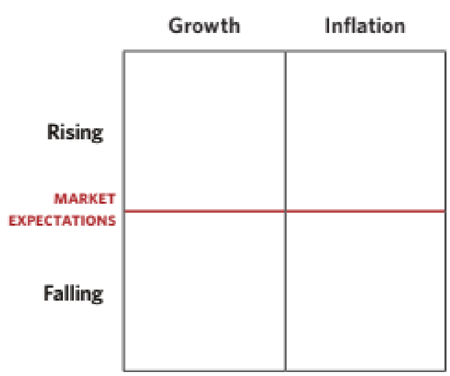


Figure 1.4.1. BW matrix of growth and inflation compared to market expectations.

Logically, 25% of a portfolio's risk would be allocated into each of these four areas, allowing the portfolio to perform in all four environments. This allocation was created by Bridgewater Associates and boils down to the reasoning that despite any shocks to the markets, value of investments can be determined by the volume of economic activity - economic growth - and the current pricing of assets - inflation. As noted in the diagram, this framework best captures the *expectations* of the market, and allows for the pricing-in of future market expectations.

ERC

The second method we are using to construct a risk parity portfolio will follow an ERC approach. We will keep the same range of asset classes for this model as to be comparable to the BW model. This range of assets helps diversify the portfolio and thus make it better suited to dealing with more economic scenarios than a portfolio containing only one type of asset. The aim of this method is to work out what proportion of the portfolio should be invested into each asset class so that the amount of risk that each asset class contributes to the overall portfolio (i.e. its risk contribution) is the same for each asset class. As a result, we would expect that the more volatile asset classes will have lower allocations than the more consistent ones, as they have a greater risk. We derive the allocations using R, a programming language for statistical computing. The ERC method is explained in further detail later in this paper.

1.5 AIMS AND GOALS

With risk parity becoming increasingly mainstream, it has never been under more scrutiny as to whether it consistently outperforms traditional portfolio allocation strategies. In this paper we will aim to use the two models presented to test the effectiveness of risk parity portfolios to lower the overall risk of a portfolio while retaining as high a return as possible. This will be done by exploring the methodology of the two methods, the ERC and BW strategies, in a more explicit proceeding and building in asset allocations in Section 2. In Section 3 we will then perform back-tests of the two portfolios to see how they would have hypothetically performed over the period 2003-2018, and discuss strengths and weaknesses accordingly.

2 METHODOLOGY

2.1 CAPITAL MARKET ASSUMPTIONS (CMAS)

While it is true that economies go through what is known as an “economic cycle” of repetitive economic conditions, investors often require a strategic long-term perspective to get a better idea of how to construct strategic allocation under future economic conditions over the next 10-15 years. As a result, private investors often rely on Capital Market Assumptions (CMAs) to give us some insight into the performance of individual asset classes in the future. CMAs have 3 key uses:

1. Developing or reviewing strategic asset allocations
2. Understanding risk-return profiles of different asset classes
3. Assessing the total risk of a certain portfolio

Once an investor has implemented an asset allocation strategy and constructed a portfolio, one of the ways to test said portfolio’s performance is to measure its returns over historical price data ranging several years (preferably decades). However, this approach has several problems. Firstly, there is no guarantee that data for all asset classes will be of equivalent length, which will make it hard to figure out an ideal time period to back-test the data for. Furthermore, the more fundamental issue with the use of historical data is the fact that this provides no guarantee that the portfolio will perform adequately well in the future.

CMAs are very useful as they are a reliable source of information for understanding future market conditions. In this paper, JPMorgan’s 2019 CMA report was used to get a correlation matrix between asset classes and measures of volatility and expected returns of each asset class. This data is derived through over 12 years of historical data adjusted for significant outliers, and any important themes and key structural changes such as the taming of the business cycle, with key insight from product specialists in each particular asset classes, a clear sign that the data released in their annual report is fairly accurate of the current and expected market conditions. This basis will hopefully provide our models with robust assumptions, thus lending more credibility to the results.

Asset Class Proxy	US Equity large cap	Commodities including Gold	U.S. Inv Grade Corp Bonds	Emerging Markets Sov Debt hedged	U.S. Long Duration Government Credit	Treasury Inflation-Indexed Security
Expected return	**6.03%**	**3.50%**	**4.67%**	**6.67%**	**4.41%**	**3.38%**
Volatility	**13.75%**	**16.25%**	**6.00%**	**9.50%**	**9.25%**	**5.25%**

Table 2.1.1. 2019 CMAs for our chosen asset classes, including long-term returns and volatility.

2.2 LEVERAGE

Leveraging is where money is borrowed in order to be invested. This allows investors to increase the amount of money they have in their portfolio, without having to use their own funds. This is especially helpful if they either don't have a lot of capital or would rather keep it for a different purpose. In theory, an investor can borrow any amount of money, so through leveraging they can also invest as much as they want to, regardless of how much they start with. This is the key advantage of leveraging, because so long as the returns of the leveraged portfolio outweigh the cost of interest from the loan, then the more leveraging an investor uses, the more amplified their returns will be.

However, this amplification will not always benefit the investor; gains are larger, but so are losses. The other disadvantage of leveraging is that interest must be paid on the loan; this cost is called the cost of carry. This means that there is the risk that if the leveraged portfolio performs badly then the returns may not offset the cost of carry, especially if the interest rate is high or volatile. Therefore, leveraging increases the risk of a portfolio, but importantly, as explained above, it also increases its potential return.

We have created two portfolios using two different methods and for each one we can apply leverage. Leveraging is very useful for constructing risk parity portfolios as the variance in our optimal portfolios is low, minimising the risk of large losses, and over time our assets should outperform cash, resulting in positive returns that are amplified by leveraging. Despite not concentrating the risk of the entire portfolio into equity like in traditional allocations, we still have to face cost of carry, which is arguably more manageable and perhaps less volatile.

2.3 BW Method

As introduced previously, the Bridgewater method is built off the assumption that the markets' expectations of two economic indicators, economic growth and inflation, best align the allocation of assets. Economic growth is chosen because it is an effective indicator of overall strength in an economy, especially the volume of demand for assets. The inflation rate has a direct implication on the discount rate, and in turn the expectations for inflation heavily determine the current pricing of assets.

Bridgewater's construction of the following 2x2 Matrix is based on 2 fundamental assumptions about investing in general:

(1) Asset price is the present value of future cash flows. It is the lump sum payment of all future cash flows, discounted at a specific rate.

(2) Assets give higher return than cash over time. This is a fundamental assumption for economic growth. This must be true generally, so that investors will put their money into assets which create value for the economy instead of saving it as currency which intrinsically does not have value.

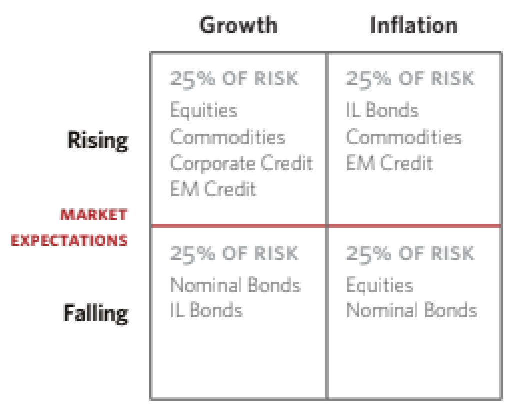


Figure 2.3.1. Risk allocation of the 6 asset classes considered based on the BW approach.

Bridgewater hypothetically places 6 assets into the 2x2 matrix shown above. We will use our interpretation of this allocation and why each asset is in its respective box.

Rising Growth contains Equities, Commodities, Corporate Credit, and Emerging Market Credit (EM Credit). Equities give you a claim on earnings, hence it is worth more when the economy and earnings are stronger. When the economy is growing, firms' profit margins are higher, they grow faster and are in a better position to take on debt, hence Corporate Credit is in this box. Furthermore, demand for commodities are higher as the demand for input factors of production are higher.

Rising Inflation contains Inflation-Linked Bonds (IL Bonds), Commodities and EM Credit. When inflation is rising, commodities can be used as a vehicle for preserving wealth over cash. IL Bonds perform in rising inflation by their very construction. EM Credit is similarly used as a vehicle of preserving value when domestic inflation negates the viability of using Corporate Credit as a vehicle for preserving value.

Falling Growth contains Nominal Bonds and IL Bonds. Nominal Bonds tend to increase in price as a result of falling interest rates as a result of Central Bank expansionary monetary policy, and are sought after in times of recession. Furthermore, in times of recession, inflation tends to rise, and so IL Bonds perform well.

Falling Inflation contains Equities and Nominal Bonds. Equities perform well in the environment of falling inflation due to the cost-cutting effect, while Nominal Bonds also perform during low inflation as their yield is typically higher than the inflation rate.

We can observe from these allocations that they are based on fundamental assumptions of the relationship between asset pricing and economic environment, and these assumptions won't change. This is arguably the advantage of this methodology over other more quantitative methods; the relationships between these asset classes change over time, and it is therefore detrimental towards the optimisation of the portfolio to rely on a mathematical relationship that changes constantly, shown in the graph below.

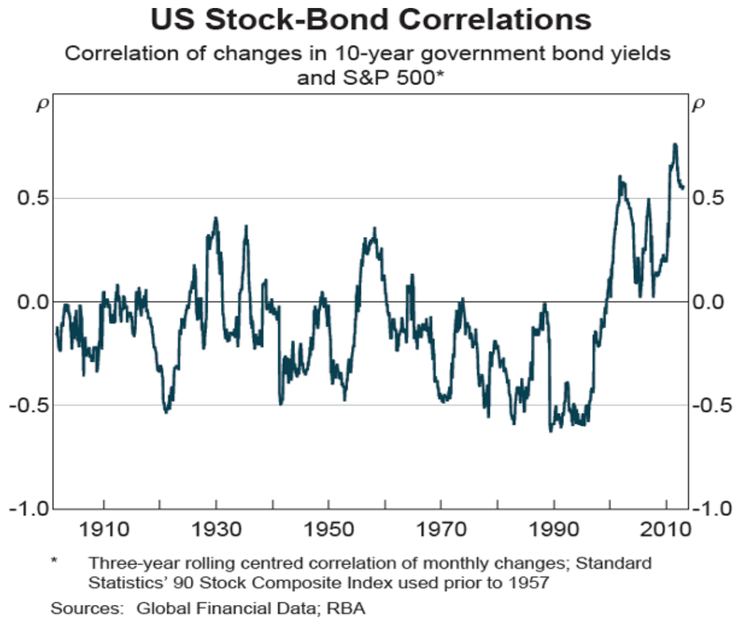


Figure 2.3.2 Correlation of changes in 10-year US government bond yield and the S&P 500.[3]

By allocating on a more fundamental basis, we avoid this issue, as well as over-optimisation of the portfolio. We include a quote from Bob Prince, Co-Chief CIO of Bridgewater Associates here to highlight the spirit of this method:

"We have seen some Risk Parity approaches that we think are overly engineered. Some are based on estimated future volatilities and correlations using recent data. Since these things change in unexpected ways, we judge this to be a problem. It is more reliable to estimate the risks and correlations of assets with a hand-held calculator and a basic understanding of asset pricing structure than by trailing quantitative measures."[4]

Construction

We design our BW model on Bridgewater's method of balancing risk. Based on excerpts of many Bridgewater publications, such as their Insights and Daily Observations, we cite the 2 principles of creating an All-Weather-style risk parity portfolio:

[3] Rankin, Shah Idil, "A History of Stock-Bond Correlations", 2014
[4] Bridgewater Associates. "Risk Parity is about Balance", 2015

"First, we increase and decrease the risk levels of all asset classes so that they have similar expected returns and risks. This provides us with a menu of return streams that have similar expected returns and risk but have different relationships with future economic environments (i.e. are lowly correlated).

Second, we select from the menu to balance assets against one another so that the portfolio doesn't have any bias to perform better or worse in any particular economic environment. We accomplish this environmental balance by holding similar exposures to assets that do well when (1) inflation rises, (2) inflation falls, (3) growth rises, and (4) growth falls."[5]

We want to take this further and replicate Bridgewater's All-Weather portfolio by applying this simple concept to their 2x2 Matrix of different assets and their bias against certain economic environments. In the following example, we will use a nominal amount of $50,000.

As seen from the BW 2x2 Matrix, an asset class can appear in two boxes of the matrix i.e. it performs well in two different economic environments. For example, Commodities perform well in both High Growth and High Inflation. To account for this, we simply divide the amount of money allocated to each box equally to each **risk-adjusted (RA**) asset within that box. For instance, in the High Inflation box, there are 3 assets, we divide the 25% amount of money equally for those 3, so roughly 8.33% (25% ÷ 3) in each RA asset for that box. Extrapolating the same step for the other 3 boxes, our risk allocation will look like the following:

[5] Bridgewater Associates. "Risk Parity is about Balance", 2015

	High Growth	**High Inflation**
High Inflation	**25% of Risk:** 6.25% in Equity 6.25% in Commodities 6.25% in Corporate Credit 6.25% in EM Credit	**25% of Risk:** 8.33% in Inflation-linked bonds 8.33% in Commodities 8.33% in EM Credit
Low inflation	**25% of Risk:** 12.5% in Nominal bonds 12.5% in Inflation-linked bonds	**25% of Risk:** 12.5% in Equity 12.5% in Nominal bonds

Table 2.3.1. Our interpretation of the BW risk allocation.

So from Table 2, across the 6 asset classes, our cross-asset risk allocation, i.e. amount of money in each RA asset class, is the following:

BW Framework	*Equities*	*Commodities*	*Corporate Credit*	*EM Credit*	*Nominal bonds*	*IL Bonds*
Risk allocation	0.1875	0.1458	0.0625	0.1458	0.25	0.2083
Volatility CMA	0.1375	0.1625	0.06	0.0950	0.0925	0.0525

Table 2.3.2 Cross-asset risk allocation of our 6-asset portfolio under the BW framework, along with their CMAs for volatility.

Based on the CMAs for each of these assets, we levered every asset to match the volatility of Commodities, as it is the most volatile asset in the set.

Row	Asset Class Proxy	US Equity large cap	Commodities	U.S. Inv Grade Corp Bonds	Emerging Markets Sov Debt hedged	U.S. Long Duration Government Credit	Treasury Inflation-Indexed Security
1	Expected return (CMA)	6.03%	3.50%	4.67%	6.67%	4.41%	3.38%
2	Volatility (CMA)	13.75%	16.25%	6.00%	9.50%	9.25%	5.25%
3	*Equalising asset risk - anchoring to highest volatility*						
4	Multiplier	1.18	1.00	2.71	1.71	1.76	3.10
5	Expected return (RA)	7.13%	3.50%	12.65%	11.41%	7.75%	10.46%
6	Volatility (RA)	16.25%	16.25%	16.25%	16.25%	16.25%	16.25%
7	Total equity	50,000					
8	Equity portion	5,925	3,900	4,525	6,670	11,740	17,240
9	Risk Allocation	18.75%	14.58%	6.25%	14.58%	25.00%	20.83%
10	**Equity Allocation**	**11.85%**	**7.80%**	**9.05%**	**13.34%**	**23.48%**	**34.48%**

Table 2.3.3. Calculations to derive the optimal asset allocation under the BW framework.

After adjusting our asset mix such that all other assets match the volatility of Commodities, we allocate the equity portion from the total amount of $50,000 based on the cross-asset risk allocation in Table 3. For example, for Equities, we allocate 11.85% × $50,000 = $5,925.

We interpret this results in 2 ways:

(1) Without access to leverage, we will allocate the capital according to Row 10.

(2) With access to leverage, we will allocate the capital according to Row 10 and then lever the portfolio to reach target risk or return.

Discussion

Overall, despite not relying on statistical assumptions about the correlation between different assets, the BW Method is not without sensitivity to assumptions.

The method relies on the fundamental assumption that the performance of each asset class has biases toward certain economic conditions. The way in which we choose the 2x2 Matrix and the asset classes in each of the 4 boxes in that matrix will also have a material effect on how much should be allocated to each asset. In this project, we choose 2 macroeconomic variables which are Inflation and Output Growth to be consistent with Bridgewater Associates' understanding of asset pricing, which may or may not be entirely accurate. Furthermore, despite not using an assumption about a covariance/correlation matrix, the method still needs some assumptions about the volatility of each asset, which is measured by standard deviation.

Moreover, the necessity of leverage in order to have risk parity across different asset classes may exclude a lot of investors, particularly retail investors using discount brokers who do not offer margin accounts, which will reduce the practicality of this method. However, the leverage can be adjusted so that investors can meet their target expected return.

2.4 ERC METHOD

For this method, we take a more analytical approach. Let us start by considering the individual risk contributions of each asset i. To achieve ERC amongst the assets, we would like the difference between risk contributions of each asset to be as close to 0 as possible. Expressed mathematically, the difference in risk contributions is of the form

$$\frac{1}{2}w^T\sum w - \frac{sum(log\ (w)\ )}{N}$$ [6],

where w is the vector containing the initial allocations to each asset class, w^T is the transpose of the vector w, $\sum$ is the covariance matrix calculated using the CMAs, $w^T\sum w$ is the overall portfolio risk, and N is the number of asset classes.

This objective function is subject to the following constraints:

- $\sum_{i=1}^{i=N} w_i = 1$ (i.e. the sum of the weightings of the asset classes needs to be 1)
- $w_i \geq 0\ for\ all\ i$ (i.e. we do not consider the possibility of short-selling for any asset class)

Recall that we want the value given by the objective to be as small as possible, because our aim is to equalise the risk contribution of each asset class. Therefore, we want to minimise this function. To do this, we used numerical methods to optimise our function in R, subject to the constraints given above, because analytical solutions are difficult to obtain, thus giving us the minimum difference in risk contributions across assets. Once this minimum difference has been found, we can then obtain the weightings that produced this value, in other words the optimal allocations subject to our assumptions. These weightings give the allocations we should use for each asset class in our risk parity portfolio and are shown in the following graph:

[6] Palomar

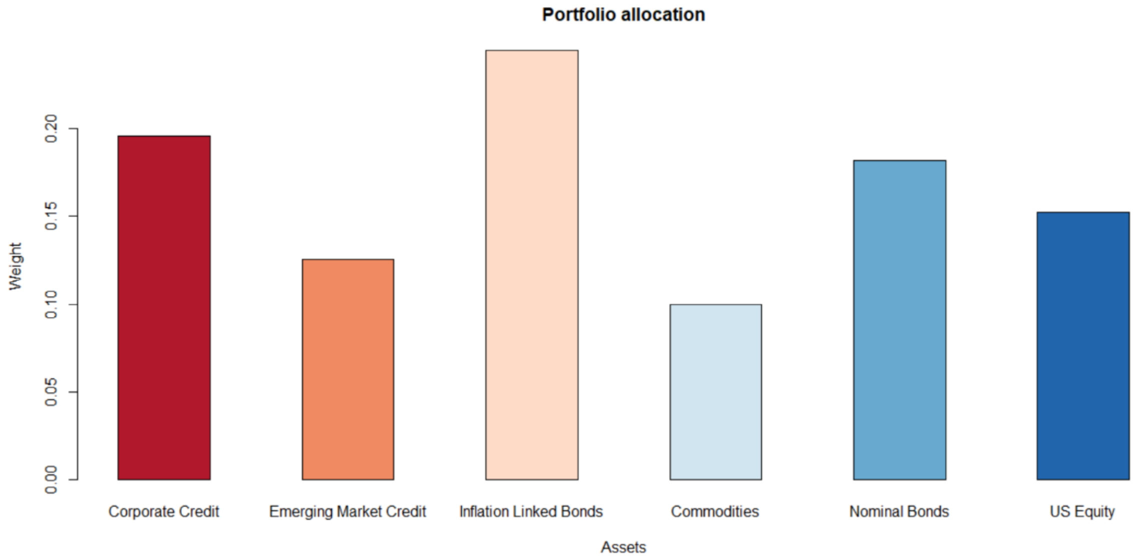


Figure 2.4.1. Optimal portfolio weightings for the 6-asset ERC model.

ERC Framework	*Corporate Credit*	*EM Credit*	*IL Bonds*	*Commodities*	*Nominal bonds*	*Equities*
Risk allocation	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667
Asset allocation	0.1960	0.1255	0.2445	0.0999	0.1820	0.1521
Comparison	*Corporate Credit*	*EM Credit*	*IL Bonds*	*Commodities*	*Nominal bonds*	*Equities*
BW allocation	0.0905	0.1334	0.3448	0.0780	0.2348	0.1185
ERC allocation	0.1960	0.1255	0.2445	0.0999	0.1820	0.1521

Table 2.4.1. Comparison of optimal solutions for both models. As above, 0.0905 represents 9.05%, for example.

Discussion

Note that compared to the BW model, the ERC model mainly differs in weightings between Corporate Credit and Inflation-Linked Bonds. This may be because historically, according to their CMAs, Commodities and US Inflation-Linked Bonds have similar risk-return profiles. As the ERC model does not heuristically assess their performances under different market conditions, allocations in both may be interchangeable to some extent.

Additionally, the ERC method relies more heavily on historical data and market assumptions due to its more quantitative approach. To reduce estimation error and to produce more robust results, we will conduct the back-tests using JPM's 2019 CMAs. While these assumptions would not have been available in 2003, we believe them to be a good enough estimate to provide a ballpark back-testing for the performance of our portfolio.

3 BACK-TESTING

In this section, we take the allocations to the past and see how the portfolios would have performed.

3.1 DATA AND METHODS

We obtain data from Bloomberg for each asset classes. Due to limitation of downloads and data available, our testing time horizon is from late 2003 to early 2019, spanning over 15 years of monthly data. However, because the Great Recession happened in 2007-08, it is still very fascinating to see how the portfolios would have performed throughout the downturn period.

Asset Class	**Proxy**	**Tracking ETF available?**
Corporate Credit	Bloomberg Barclays US Corporate Total Return Value Unhedged USD	Yes
Emerging market credit	J.P. Morgan EMBI Global Total Return Index	No, but there are very close ETF that moves similarly.
Inflation-linked bonds	iShares TIPS Bond ETF	Yes (We could not find any reliable index so went straight to the ETF security).
Commodities	Bloomberg Commodity Index	Yes
Nominal bonds	Bloomberg Barclays US Treasury Total Return Unhedged USD	Yes
Equities	S&P 500 Index	Yes

Table 3.1.1. Asset classes and proxies used for back-test.

Our method is simple. Using Excel, we subject the amount allocated to the price changes of each asset class and balance our portfolio to maintain the allocation every quarter. For the ERC portfolios, we do not adjust our variance-covariance assumptions, as we believe that the CMAs are robust enough as estimates for our entire back-test window. We compare both portfolios to the S&P 500, a 60/40 allocation, and a mean-variance optimised portfolio (MVO) with a target annualised return comparable to that of our models, subject to the same variance-covariance assumptions and expected return CMAs, constructed with MatLab.

3.2 Results

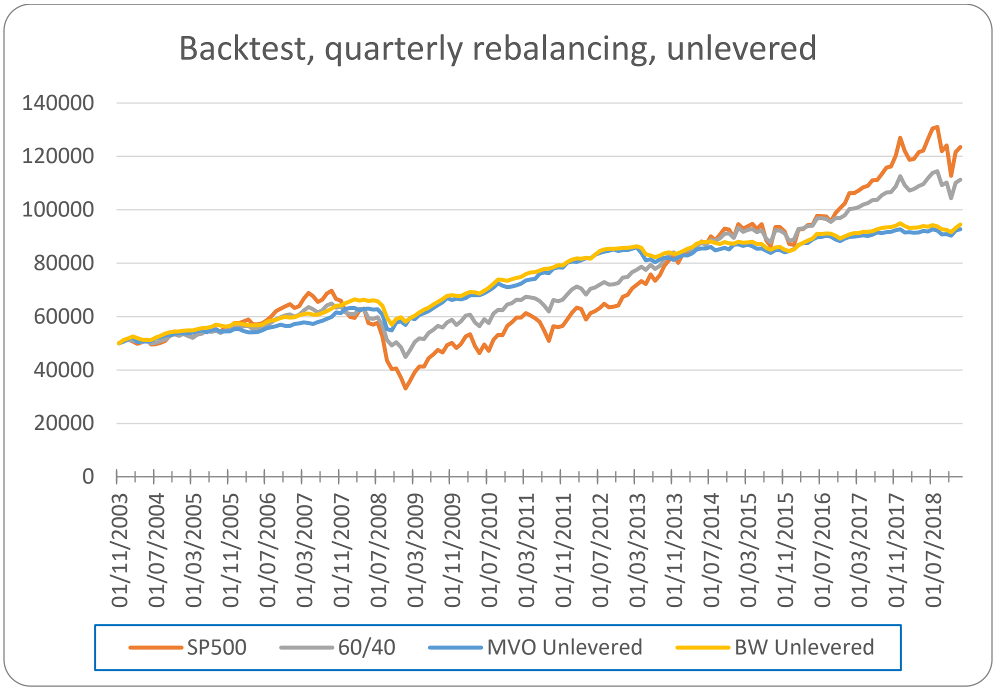


Figure 3.2.1. BW unlevered portfolio back-test performance against the S&P 500, a 60/40 allocation, and a MVO portfolio with similar annualised returns from 2003-2019.

The BW unlevered allocation is derived from Row 10 of Table 2.3.3. The MVO allocations are as follows:

MVO Allocations	*Equities*	*Commodities*	*Corporate Credit*	*EM Credit*	*Nominal bonds*	*IL Bonds*
Risk allocation	0.2754	0	0.3065	0	0	0.4182
Equity Allocation	0.0870	0	0.2218	0	0	0.6912

Table 3.2.1. Risk and equity allocations of the MVO portfolio with a target return comparable to the risk parity portfolios using the same variance-covariance assumptions. As above, 0.0870 represents 8.70%, for example.

As Figure 3.2.1 shows, the BW portfolio is significantly less volatile and more resilient when compared to the S&P 500 and the traditional 60/40 allocation, especially from 2007-2011. It performs very similarly to the MVO portfolio, but has a few advantages: Firstly, its risk to return ratio is better[7], which is the main goal of our risk parity portfolio. Additionally, the MVO model has a disproportionate allocation in Inflation-Linked bonds and no allocations in three of the six asset classes, as seen from Table 3.2.1. This gives it around a 42% risk contribution to the portfolio,

[7] See summary statistics in Table 3.2.2.

31% to Corporate Credit, and 28% to Equities. Although this particular MVO portfolio performs well over our back-test window, one can see from the asset allocations and corresponding risk contributions that it is not diversified in terms of either, and thus may underperform under other economic conditions.

We also see that despite our best efforts, performance of the BW portfolio still dips, albeit not considerably, during the financial crisis, as all asset classes underperformed cash during that time period. However, the decrease in volatility also means it underperforms its counterparts during bull markets, and thus we consider a levered version of the BW portfolio, where we lever it to have the same annualised volatility as the S&P 500 over the back-test window.

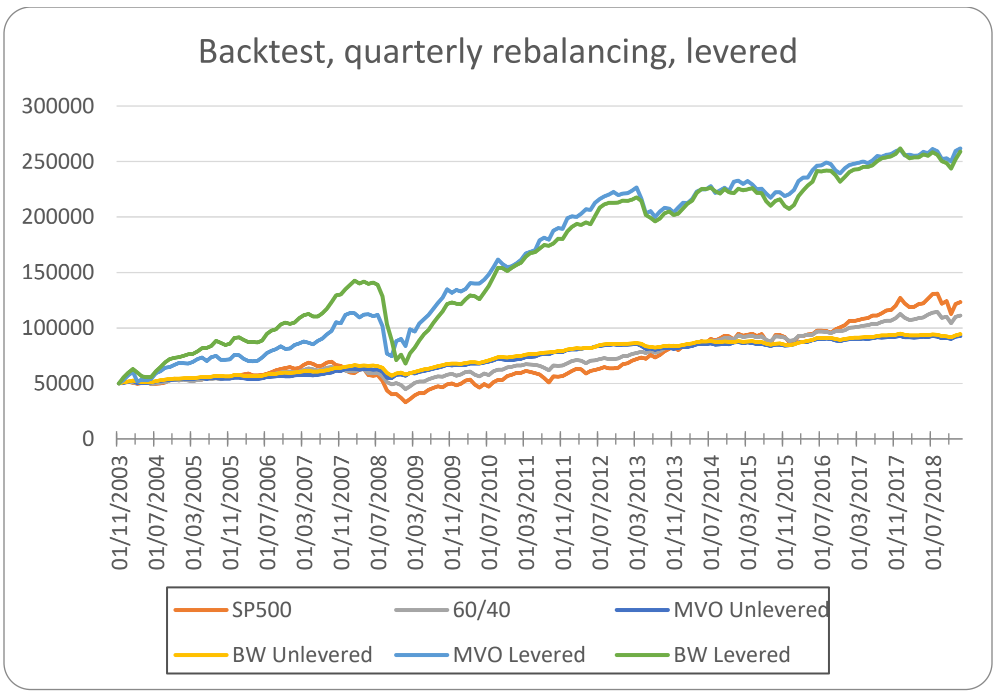


Figure 3.2.2. BW 5.7x levered portfolio back-test performance against the S&P 500, a 60/40 allocation, and a 4.7x levered MVO portfolio from 2003-2019.

After goal-seeking in Excel, we find that a roughly 5.7x lever will produce the same annualised volatility as the S&P 500. The result is optimistic and shows that the levered BW portfolio outperformed the S&P 500 significantly on a cumulative basis. The 4.7x levered MVO portfolio, which also has the same annualised volatility as the S&P 500, produces similar results. However, the back-test is imperfect, meant only for illustration purposes and does not guarantee future returns. Due to several constraints, we have not been able to formally include a number of factors that could potentially affect the performance such as cost of carry, liquidity and so on. It is worth mentioning however that through cursory testing, the cost of carry does not meaningfully decrease our returns, and is thus somewhat negligible in this case.

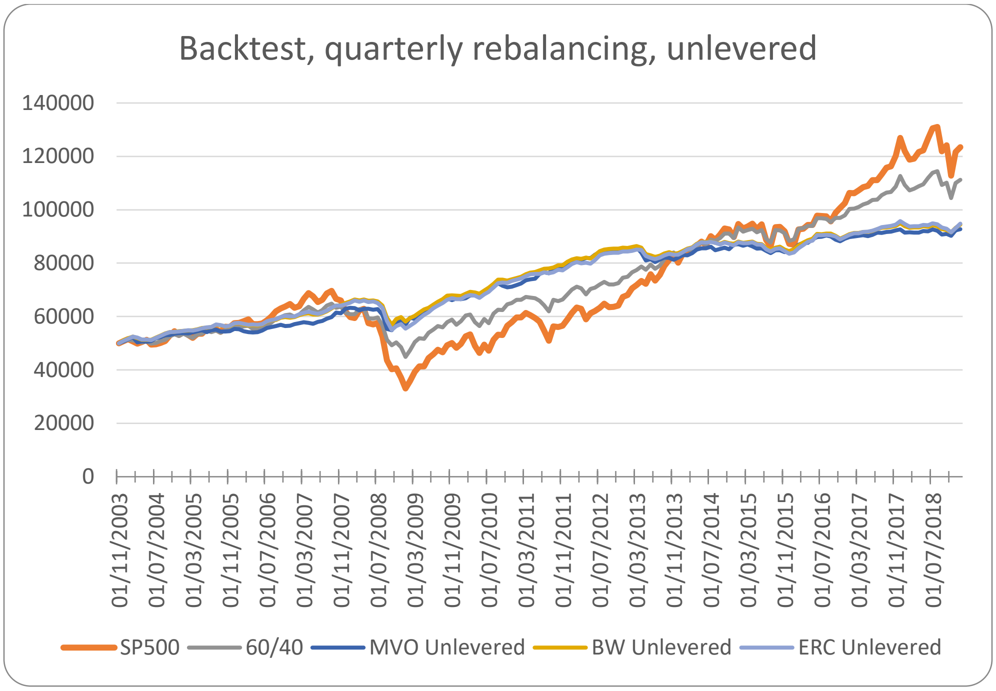


Figure 3.2.3 ERC unlevered portfolio back-test performance against the S&P 500, 60/40 allocation, a MVO portfolio, and the BW portfolio from 2003-2019.

Figure 3.2.3 shows the unlevered ERC portfolio overlaid on Figure 3.2.1. We can see that its performance over our back-test window is almost identical to that of the BW model; indeed, their summary statistics are also very similar. This may suggest that both portfolios that both portfolio allocations are interchangeable, however as Figure 3.2.4 illustrates, this is only the case for the respective unlevered versions.

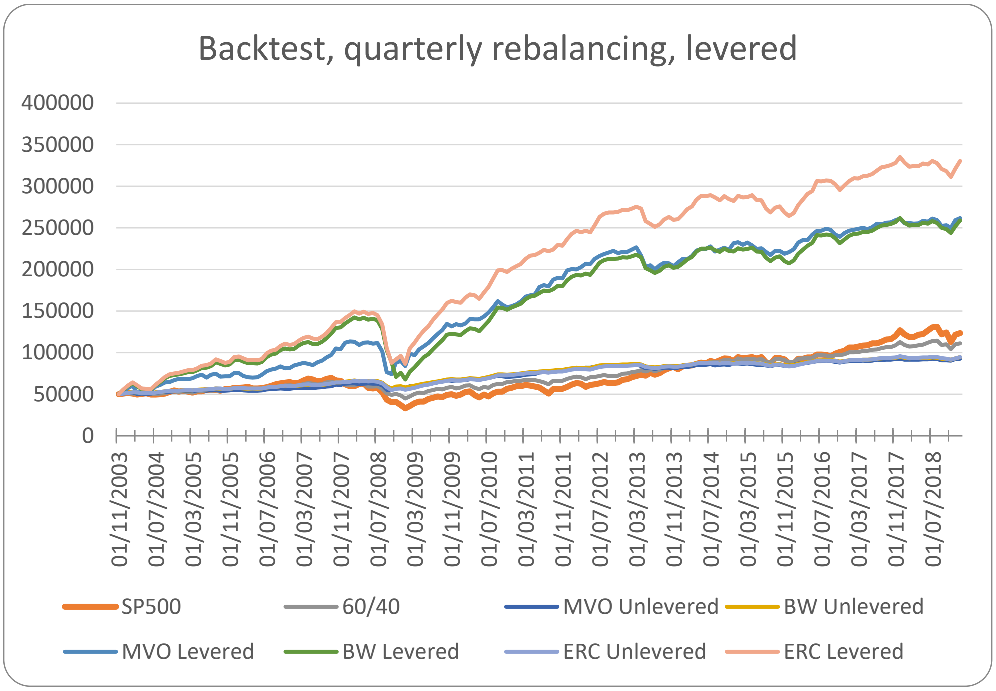


Figure 3.2.4 ERC 6.2x levered portfolio back-test performance against the S&P 500, a 60/40 allocation, the MVO portfolios, and the BW portfolios from 2003-2019.

We see in Figure 3.2.4 that after the financial crisis, the 6.2x levered ERC portfolio outperforms the levered MVO portfolio, and also the levered BW portfolio. This may be due to the increased allocation in Corporate Credit and a decreased allocation in Commodities, compared to its BW counterpart, whose performances happened to favour the ERC model during the back-test window. The ERC portfolio is also inherently slightly more volatile, which may have contributed to it capturing this upward momentum better. However, the aim of both models is to reduce risk, especially from extrinsic factors, and thus this shows that the BW construction may be more resilient towards external changes.

Portfolio	**Return (CAGR)**	**Annualised Risk**	**Annualised Return/Risk**	**Correlation with S&P 500**
S&P 500	5.98%	**13.57%**	0.44	**1**
60/40	5.30%	8.35%	0.64	0.98
MVO Unlevered	4.08%	4.73%	0.86	0.42
MVO Levered	11.12%	**13.57%**	0.82	0.38
BW Unlevered	4.07%	4.02%	**1.01**	0.46
BW Levered	10.62%	**13.57%**	0.78	0.46
ERC Unlevered	4.10%	4.42%	0.93	0.46
ERC Levered	**12.41%**	**13.57%**	0.91	0.46

Table 3.2.2. Summary statistics of the respective portfolios, showing annualised return and risk, their ratio, as well as correlation with the S&P 500. Largest values of each column are highlighted.

4 CONCLUDING REMARKS

With increasing uncertainty in our economic and political landscape, a deeper dive into return-free strategies seems to be warranted. In this paper, we considered the background and intuition of risk parity and why it has gained in popularity. We explored two strategies for constructing risk parity portfolios and employed them to create corresponding 6-asset portfolios which aim to produce returns independent of extrinsic factors. We also applied leverage to amplify returns given our optimal allocations, and subsequently back-tested their performance against that of the S&P 500 and other allocation strategies for the years 2003-2019.

The performance of our portfolios was promising. Returns were significantly less volatile than traditional allocations and outperformed noticeably during the financial crisis. In addition, if we leverage the risk parity portfolios to have the same overall level of risk as the S&P 500, we see that our portfolios provide much greater returns. Furthermore, even though the unlevered risk parity portfolios were outperformed by the S&P 500 in times where there was stronger economic growth, this is not unexpected as this is when equities perform best. The main result we take away from this is that in the last 15 years our risk parity portfolios can achieve much higher return for the same amount of volatility as equities-heavy portfolios, hence it is a more balanced approach.

There is no guarantee that our portfolios will perform similarly in the future, or that past economic conditions will repeat themselves. However, the main intuition behind risk parity is to construct balances portfolios with stable returns that will stand the test of time; the point is not to predict the future, but rather to weather the unpredictable. We believe the potential rewards of risk parity portfolios that we have recognised throughout this paper to be perennial, and we fully expect their popularity to continue rising, and that the methods used to construct them will be further improved on in the future to extract their full potential.

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